B.A./B.Sc. 5th Semester (Honours) Examination, 2023 (CBCS)

Subject : Mathematics

Course : BMH5CC12

(Mechanics-I)

Time: 3 Hours **Full Marks: 60**

The figures in the margin indicate full marks.
Candidates are required to give their answers in their own words as far as practicable.

Notation and symbols bear usual meaning.

1. Answer *any ten* questions from the following: 2×10=20

(a) Define astatic equilibrium for a system of coplanar forces.

(b) State the principle of virtual work for a particle.

(c) Obtain the centre of gravity of a semicircular arc revolving about the bounding diameter.

(d) Obtain the degree of freedom of a rigid body which is fixed in space at its any three non collinear points.

(e) Define Poinsot's central axis in a system of forces acting on a rigid body.

(f) A particle is executing Simple Harmonic Motion (S.H.M.) such that its period of oscillation is π seconds. If its maximum acceleration is 12 ft/sec^2, find its amplitude.

(g) An insect crawls at a constant rate u along the spoke of a cartwheel of radius a. The cart is moving with velocity v. Calculate the acceleration along and perpendicular to the spoke. 1+1

(h) Find the velocity of an artificial satellite of the earth, given $g = 9{\cdot}8$ metres/sec^2, radius of the earth = $6{\cdot}4 \times 10^8$ metres. (Assuming that the satellite is moving very close to the surface of the earth).

(i) If the path of a particle be a circle with radius a, find its radial and cross-radial accelerations.

(j) Prove that the particle moves at right angle to the radius vector at an apse.

(k) Prove that a planet has only a radial acceleration towards the Sun.

(l) If P, Q, R act along three non-intersecting edges of a cube, find the central axis.

(m) What is angular momentum? Using the concept of angular momentum prove the relation $h = v.p$, where the letters have their usual meaning. 1+1

(n) A particle is moving along the curve of an equiangular spiral under the force P to the pole. Find the law of force.

(o) What do you mean by constraint of a dynamical system? Give an example.

2. Answer *any four* questions from the following: 5×4=20

(a) (i) What is the coefficient of friction in motion of a body over the surface?

(ii) Show that for equilibrium, the resultant reaction can never make with the normal an angle greater than the angle of friction. 2+3

(b) A particle is projected from the earth's surface vertically upwards with a velocity v. If h and H are the greatest heights attained by the particle moving under uniform and variable acceleration respectively, show that $\frac{1}{h} - \frac{1}{H} = \frac{1}{R}$ where R is the radius of the earth.

(c) Obtain the components of velocity and acceleration of a particle along and perpendicular to the radius vector to it from a fixed origin.

(d) If a planet was suddenly stopped in its orbit, supposed circular, show that it will fall into the sun in a time which is $\frac{\sqrt{2}}{8}$ times the period of the planet's revolution.

(e) The length AB and CD of the sides of a rectangle $ABCD$ are $2a$ and $2b$; show that the inclination of one of the principal axes with AB at A is $\frac{1}{2}\tan^{-1}\left(\frac{3ab}{2(a^2-b^2)}\right)$.

(f) A uniform rod is held at an inclination α to the horizon with one end in contact with a horizontal table whose coefficient of friction is μ. If it be then released, show that it will commence to slide if $\mu < \frac{3\sin\alpha\cos\alpha}{1+3\sin^2\alpha}$.

3. Answer *any two* questions from the following: 10×2=20

(a) (i) A uniform chain of length l is to be suspended from two points A and B in the same horizontal line so that either terminal tension is n times of that at the lowest point. Show that the span AB must be $\frac{l}{\sqrt{n^2-1}}\log_e\left(n+\sqrt{n^2-1}\right)$.

(ii) Show that the momental ellipsoid at the centre of an elliptic plate is $\frac{x^2}{a^2}+\frac{y^2}{b^2}+z^2\left(\frac{1}{a^2}+\frac{1}{b^2}\right) = \text{constant}$. 5+5

(b) (i) A particle is projected with velocity u at an inclination α above the horizontal in a medium whose resistance per unit mass is k times the velocity. Show that its direction will again make an angle α below the horizontal after a time $\frac{1}{k}\log\left(1+\frac{2ku}{g}\sin\alpha\right)$.

(ii) A particle moves in a straight line from rest under an attractive force (acceleration) $\mu \times (\text{distance})^{-2}$ directed towards a fixed point on the line, where μ is a constant. Show that if the initial distance is $2a$, then the distance will be 'a' after a time $\left(\frac{\pi}{2}+1\right)\left(\frac{a^3}{\mu}\right)^{\frac{1}{2}}$. 5+5

(c) (i) Three forces act along the straight lines $x=0, y-z=a$; $y=0, z-x=a$; $z=0, x-y=a$. Show that they cannot reduce to a couple. Prove also that if the system reduces to a single force its line of action must lie in the surface
$x^2+y^2+z^2-2yz-2zx-2xy = a^2$.

(ii) A particle moves under a central acceleration $\frac{\mu}{r^3}$. It is projected from an apse at a distance a from the centre of force with a velocity equal to $\sqrt{2}$ times the velocity in a circle at the same distance, show that the path is $r\cos\left(\frac{1}{\sqrt{2}}\theta\right) = a$. 5+5

(d) (i) Define catenary of uniform strength and deduce its equation in cartesian form.

(ii) A heavy uniform rod AB of length $2a$, rests with its ends in contact with two smooth inclined plane of inclination α and β to the horizon. Prove by principle of virtual work that $\tan\theta = \frac{1}{2}(\cot\alpha - \cot\beta)$. (2+3)+5

———